

Fourierova transformace distribucí

$$(1): \quad f \in L^1(\mathbb{R})$$

$$\mathcal{F}(f) = \int_{\mathbb{R}} f(x) \exp\{-2\pi i x \xi\} dx$$

$$(2): \quad A \in \mathbb{R}^{m \times m}, \text{ poz. definitní, symetrická}$$

$$\mathcal{F}(\exp\{-(Ax, x)\}) = \frac{(\sqrt{\pi})^m}{\sqrt{|\det A|}} \exp\{-\pi^2(A^{-1}\xi, \xi)\}$$

$$(3): \quad \delta \in S'$$

$$\mathcal{F}(\delta) = 1$$

$$(4):$$

$$\mathcal{F}(1) = \delta$$

$$(5): \quad x^n \in S'$$

$$\mathcal{F}(x^n) = \frac{1}{(-2\pi i)^n} \delta^n(\xi)$$

$$(6):$$

$$\mathcal{F}(\delta^{(n)}) = (2\pi i)^n \xi^n$$

$$(7): \quad b \in \mathbb{C}$$

$$\mathcal{F}(\exp(2\pi i bx)) = \delta_b$$

$$(8): \quad b \in \mathbb{C}$$

$$\mathcal{F}(\sin(2\pi bx)) = \frac{1}{2i}(\delta_b - \delta_{-b})$$

$$(9): \quad b \in \mathbb{C}$$

$$\mathcal{F}(\cos(2\pi bx)) = \frac{1}{2}(\delta_b + \delta_{-b})$$

$$(10): \quad b \in \mathbb{C}$$

$$\mathcal{F}(\sinh(2\pi bx)) = \frac{1}{2}(\delta_{-ib} - \delta_{ib})$$

$$(11): \quad b \in \mathbb{C}$$

$$\mathcal{F}(\cosh(2\pi bx)) = \frac{1}{2}(\delta_{-ib} + \delta_{ib})$$

$$(12): \quad x \in \mathbb{R} \quad \lambda \in \mathbb{C}$$

$$\mathcal{F}\left(\frac{x_+^\lambda}{\Gamma(\lambda+1)}\right) = e^{-i(\lambda+1)\frac{\pi}{2}} (2\pi)^{-\lambda-1} (\xi - i0)^{-\lambda-1}$$

$$(13): \quad \mathcal{F}(x_+^n) =$$

$$= (2\pi i)^{-n-1} n! \xi^{-n-1} + \frac{1}{2} (2\pi i)^{-n} (-1)^{-n} \delta^{(n)}$$

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$$(14):$$

$$\mathcal{F}(\theta(x)) = \mathcal{F}(x_+^0) = \frac{1}{2\pi i} \xi^{-1} + \frac{1}{2} \delta$$

$$(15):$$

$$\mathcal{F}\left(\frac{x_-^\lambda}{\Gamma(\lambda+1)}\right) = e^{i(\lambda+1)\frac{\pi}{2}} (2\pi)^{-\lambda-1} (\xi + i0)^{-\lambda-1}$$

$$(16): \quad |x|^\lambda = x_+^\lambda + x_-^\lambda, \quad \lambda \neq -1, -2, -3, \dots$$

$$\mathcal{F}(|x|^\lambda) = -2\Gamma(\lambda+1)(2\pi)^{-\lambda-1} \sin\left(\frac{\pi}{2}\lambda\right) |\xi|^{-\lambda-1}$$

$$(17): \quad \mathcal{F}(|x|^\lambda \operatorname{sign} x) =$$

$$= -2i(2\pi)^{-\lambda-1} \Gamma(\lambda+1) \cos\left(\frac{\pi}{2}\lambda\right) |\xi|^{-\lambda-1} \operatorname{sign} \xi$$

$$(18): \quad m \in \mathbb{N}; \quad \mathcal{F}(x^{-m}) =$$

$$= \begin{cases} (-1)^{\frac{m+1}{2}} i\pi (2\pi)^{m-1} |\xi|^{m-1} \frac{\operatorname{sign} \xi}{(m-1)!} & m \text{ liché} \\ (-1)^{\frac{m}{2}} \frac{|\xi|^{m-1} \pi (2\pi)^{m-1}}{(m-1)!} & m \text{ sudé} \end{cases}$$

$$(19):$$

$$\mathcal{F}(x^{-1}) = -i\pi \operatorname{sign} \xi$$

$$(20):$$

$$\mathcal{F}(x^{-2}) = -|\xi| 2\pi^2$$

$$(21):$$

$$\mathcal{F}((x+i0)^\lambda) = \frac{\xi_+^{-\lambda-1}}{\Gamma(-\lambda)} \exp\{i\lambda\frac{\pi}{2}\} (2\pi)^{-\lambda}$$

$$(22):$$

$$\mathcal{F}((x-i0)^\lambda) = \frac{\xi_-^{-\lambda-1}}{\Gamma(-\lambda)} \exp\{-i\lambda\frac{\pi}{2}\} (2\pi)^{-\lambda}$$

$$(23): \quad \forall \lambda \in \mathbb{C}$$

$$\mathcal{F}\left(\frac{r^\lambda}{\Gamma(\frac{\lambda+N}{2})}\right) = \frac{\rho^{-\lambda-N}}{\Gamma(-\lambda/2) \pi^{\lambda+N/2}}$$